

# Finite Element Analysis by using Three-Dimensional Laminate core Elements

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**Abstract** — This paper presents finite element analysis by using three-dimensional laminated core elements. The number of finite elements can be reduced by using three-dimensional laminated core elements. This element is based on the other hexahedral element, which has two material regions, the steel sheet and surface coating regions. By solving a fundamental model usefulness of this method to the three dimensions analysis is shown. The final goal of this study is the three-dimensional analysis considering lamination of core materials and their vector magnetic. The knowledge obtained in the numerical simulations properties is reported in this paper.

## I. INTRODUCTION

To think better electrical machinery and apparatus, it is important to know how to suitably select and use the magnetic properties of core materials such as electrical steel steels. it is therefore important to develop the modeling of laminated core considering the stacking factor to analyze practical electrical machinery and apparatus.

Generally, in conventional finite element analysis of electromagnetic problems including laminated electrical steel sheets, the magnetic material are usually modeled as a non-laminated core or yoke. Electrical steel sheet has magnetic anisotropy. Two-dimensional vector magnetic properties are proposed as the method that magnetic anisotropy is considered. Analysis that we apply this method and considered magnetic anisotropy may be produced.

## II. DEFINITION OF 3D LAMINATED CORE

### A. Formulation

Fig.1 shows a element considering the insulator.  $S^{(e)}$  is the steel region,  $\Omega^{(e)}$  is insulator area, and  $\delta$  is the thickness of the insulator. When the thickness of the insulator is very thin in comparison with the one of the steel,  $A_9, A_{10}, A_{11}, A_{12}$  can be approximated as,

$$A_9 = A_1, A_{10} = A_2, A_{11} = A_3, A_{12} = A_4 \quad (1)$$

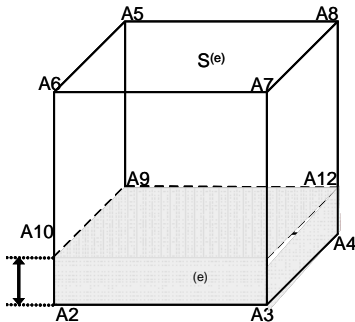


Fig.1 Element considering the insulator

Generally, the Garelkin's equation for three-dimensional problems is written as follows

$$\nu \int_V \left( \frac{\partial N_i}{\partial x} \frac{\partial A_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial A_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial A_j}{\partial z} \right) dV = \int_V N_i J dV \quad (2)$$

where,  $\nu$  is magnetic reluctivity,  $N_i$  the shape function of the hexahedral element,  $J$  the exciting current density,  $A$  the magnetic vector potential.<sup>[1]</sup>

The coefficient matrix of the steel region is supposed as  $S$ , and that of the insulator area is can be written as.

$$S_{ij} = \nu \int_V \left( \frac{\partial N_i}{\partial x} \frac{\partial A_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial A_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial A_j}{\partial z} \right) dV \quad (3)$$

$$Q_{ij} = \delta \int_S \sum_{j=1}^4 \left( \frac{\partial M_i}{\partial x} \frac{\partial M_j}{\partial x} + \frac{\partial M_i}{\partial y} \frac{\partial M_j}{\partial y} \right) dS d\Omega \quad (4)$$

$\delta$  is the thickness of the insulator, and  $M_i$  is the two-dimensional interpolation function. Total coefficient matrix is written as

$$L_{ij} \begin{Bmatrix} A_1 \\ \vdots \\ A_8 \end{Bmatrix} = S_{ij} \begin{Bmatrix} A_1 \\ \vdots \\ A_8 \end{Bmatrix} + Q_{ij} \begin{Bmatrix} A_1 \\ \vdots \\ A_4 \end{Bmatrix} \quad (5)$$

$L_{ij}$  is the elemental coefficient matrix

$$S_{ij} = \frac{bc\alpha_i\alpha_j}{8a} \left( 1 + \frac{\beta_i\beta_j}{3} \right) \left( 1 + \frac{\gamma_i\gamma_j}{3} \right) + \frac{ac\beta_i\beta_j}{8b} \left( 1 + \frac{\alpha_i\alpha_j}{3} \right) \left( 1 + \frac{\gamma_i\gamma_j}{3} \right) + \frac{ab\gamma_i\gamma_j}{8c} \left( 1 + \frac{\alpha_i\alpha_j}{3} \right) \left( 1 + \frac{\beta_i\beta_j}{3} \right) \quad (6)$$

$a, b$  and  $c$  are length of the side of the element

$$Q_{ij} = \delta \nu \frac{ba\alpha_i\alpha_j}{4a} \left( 1 + \frac{\beta_i\beta_j}{3} \right) + \delta \nu \frac{a\beta_i\beta_j}{4b} \left( 1 + \frac{\alpha_i\alpha_j}{3} \right) \quad (7)$$

### B. Two-dimensional Magnetic Properties

The magnetic flux density  $\mathbf{B}$  and the magnetic field strength  $\mathbf{H}$  in anisotropic materials are not parallel when the rotating magnetic field is applied to materials and the alternating magnetic field is applied with an angle  $\theta_b$  from the easy axis. Such magnetic properties have strong nonlinearity as a function

of the maximum flux density  $B_{max}$  and the inclination angle  $\theta_B$ . The differential type E&S modeling can express both the alternating and rotating hysteresis as the relationship between  $B$  and  $H$  during one period. The modeling is defined with the following equations.

$$H_x = \nu_{xr} (B_{max}, \theta_B, \alpha, \tau) B_x(\tau) + \nu_{xi} (B_{max}, \theta_B, \alpha, \tau) \frac{\partial B_x(\tau)}{\partial \tau} \quad (1)$$

$$H_y = \nu_{yr} (B_{max}, \theta_B, \alpha, \tau) B_y(\tau) + \nu_{yi} (B_{max}, \theta_B, \alpha, \tau) \frac{\partial B_y(\tau)}{\partial \tau} \quad (2)$$

where,  $\nu_{xr}$  and  $\nu_{yr}$  are called the magnetic reluctivity coefficients, and then  $\nu_{xi}$  and  $\nu_{yi}$  are called the magnetic hysteresis coefficients. The magnetic reluctivity coefficients and the magnetic hysteresis coefficients are altered in time as a periodical function. The variable  $\tau$  is in a range from zero to  $2\pi$ . The coefficient  $\alpha$  is called the axis ratio, which can be defined as the ratio of the minimum flux density and the maximum flux density. Figs. 2 (a) and (b) show the definitions of the alternating flux condition and the rotating flux condition, respectively. The precise circular rotating flux means that  $\alpha$  is one, and the alternating flux condition means  $\alpha$  is zero [4][5].

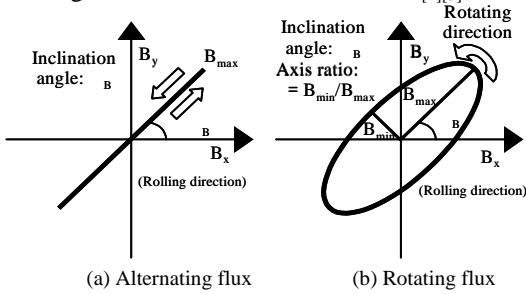


Fig. 2 Definition of the alternating and rotating flux.

### III. ANALYSIS CONDITIONS

#### A. Analyzed Model

Fig.3 shows Benchmark model of laminated iron core. DC of 3000[AT] is flowing to excite coil evenly. The core is constructed by laminating 200 steel sheets (JIS grade: 50A700) in the x-direction. The thickness of the steel sheet is 0.5 mm and the space factor is set to be 0.96. As magnetic characteristic of the steel sheets, we neglect hysteresis and a isotropy.

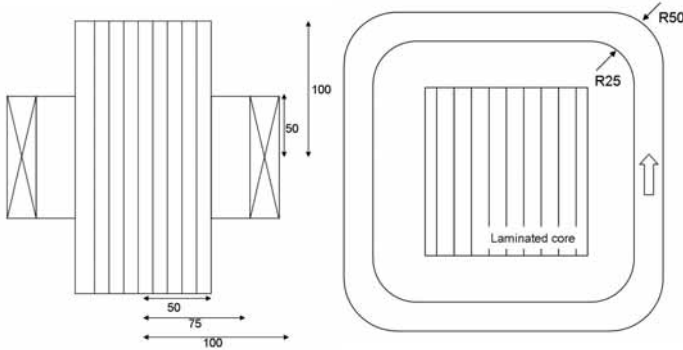


Fig.3 Benchmark model of laminated iron core

TABLE I  
ANALYZED CONDITION

Current density	1500000 [A/m <sup>2</sup> ]
Laminated core	200 [pieces]
Number of turn	40 [turns]
Electrical steel sheet	50A700
Number of nodes	24300
Number of elements	26908

#### B. Results

The analyzed results by using three-dimensional laminated core elements were compared with that of the measurement value of the same model.

The upper and side of the evaluated points are shown in Fig. 4. The points are distributed in the air region laminated core.

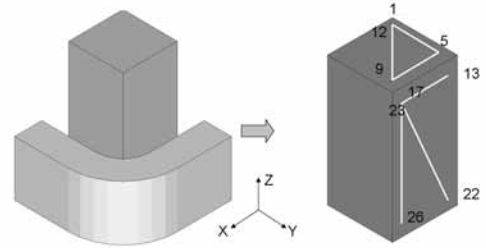


Fig4, Evaluated points of Benchmark model

### IV. CONCLUSION

This paper presented the three dimensional analysis by using three-dimensional laminated core elements. We report about the appropriateness of the result with this paper. Three-dimensions analysis that considered magnetic anisotropy are produced detailed analyses of many electrical machinery and apparatus

### V. REFERENCES

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